TEMPERATURE FIELD CLOSE TO THE BOUNDARY OF TWO DIFFERENT THROTTLING LIQUIDS

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A simplified mathematical model of the thermal field close to the boundary of two different throttling liquids is considered.

The study of thermal phenomena when liquids are throttled has given rise to the possibility of developing new methods for making physical investigations. There is therefore a need for a further development of the theory of thermal processes during throttling [1]. In this paper we consider the problem of the thermal field close to the boundary of two different throttling liquids. We assume that: 1) the part played by convective heat transfer is considerably greater than the part played by conductive heat transfer, so that the effect of thermal conduction along the throttling path can be neglected; 2) the boundary the between the throttling liquids is plane; and 3) the transients which occur when the pressure is building up can be neglected, i.e., $\partial P/\partial t = 0$.

Preliminary calculations show that, according to condition 1, the model considered below is applicable for throttling speeds $u > \lambda/c\rho R$. For values of the heat capacity $c = 2000$ J/kg·deg K, a density $\rho = 800$ kg/m^3, $\lambda = 1.7$ W/m·deg K and dimensions of the throttling region $R = 1$ m, we obtain $u > 10^{-6}$ m/sec.

The mathematical formulation of the problem in dimensionless variables has the form

$$\frac{\partial T_1}{\partial \mathrm{Fo}} + u\left[\frac{\partial T_1}{\partial \varkappa} + \varepsilon_{\mathrm{I}} \frac{\partial P_1}{\partial \varkappa}\right] = a^2 \frac{\partial^2 T_1}{\partial z^2}; \quad \begin{matrix} z > 0; \\ \mathrm{Fo} > 0; \end{matrix} \quad \varkappa > 0, \tag{1}$$

$$\frac{\partial T_2}{\partial \mathrm{Fo}} + \frac{\partial T_2}{\partial \varkappa} + \varepsilon_{\mathrm{II}} \frac{\partial P_2}{\partial \varkappa} = \frac{\partial^2 T_2}{\partial z^2}; \quad \begin{matrix} z < 0; \\ \mathrm{Fo} > 0; \end{matrix} \quad \varkappa > 0, \tag{2}$$

$$T_{1,2}|_{\mathrm{Fo}=0} = 0; \; T_{1,2}|_{\varkappa=0} = 0, \tag{3}$$

$$T_1|_{z=0} = T_2|_{z=0}; \quad \left.\frac{\partial T_1}{\partial z}\right|_{z=0} = \lambda \left.\frac{\partial T_2}{\partial z}\right|_{z=0}. \tag{4}$$

Here $\mathrm{Fo} = a_2 t/R^2$; $\varkappa = \bar{x} a_2/u_2 R^2$; $z = \bar{z}/R$; $a^2 = a_1/a_2$; $u = u_1/u_2$; $\lambda = \lambda_2/\lambda_1$. $T_{1,2}(z, \varkappa, \mathrm{Fo})$ is a bounded function.

In two-dimensional Laplace–Carson image space

$$v_{1,2} = sq \int_0^\infty d\mathrm{Fo} \int_0^\infty \exp\left[-(s\mathrm{Fo} + q\varkappa)\right] T_{1,2}(\mathrm{Fo}, \varkappa, z)\, d\varkappa \tag{5}$$

the problem takes the form

$$a^2 \frac{d^2 v_1}{dz^2} = (s + uq) v_1 + u\varepsilon_{\mathrm{I}} q P_1(q); \; z > 0, \tag{6}$$

$$\frac{d^2 v_2}{dz^2} = (s + q) v_2 + \varepsilon_{\mathrm{II}} q P_2(q); \; z < 0, \tag{7}$$

$$v_1|_{z=0} = v_2|_{z=0}; \quad \left.\frac{\partial v_1}{\partial z}\right|_{z=0} = \lambda \left.\frac{\partial v_2}{\partial z}\right|_{z=0}. \tag{8}$$

The solutions of (6) and (7), taking into account the boundedness, can be represented in the form

$$v_1 = -\frac{u\varepsilon_{\mathrm{I}} q P_1(q)}{s + uq} + C_1 \exp\left(-\sqrt{s + uq}\,\frac{z}{a}\right); \quad z > 0, \tag{9}$$

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$$v_2 = -\frac{\varepsilon_{11} q P_2(q)}{s+q} + C_2 \exp(-\sqrt{s+q}\,|z|);\; z<0. \tag{10}$$

From the boundary conditions (8) we obtain a system of two equations for determining C_1 and C_2.

The solution of this system has the form

$$C_1 = \left[\frac{u\varepsilon_1 q P_1(q)}{s+uq} - \frac{\varepsilon_{11} q P_2(q)}{s+q}\right] \frac{1}{1+\frac{\sqrt{s+uq}}{\lambda a\sqrt{s+q}}}, \tag{11}$$

$$C_2 = \left[\frac{\varepsilon_{11} q P_2(q)}{s+q} - \frac{u\varepsilon_1 q P_1(q)}{s+uq}\right] \frac{1}{1+\frac{\lambda a\sqrt{s+q}}{\sqrt{s+uq}}}. \tag{12}$$

Substituting (12) and (11) into (9) and (10), respectively, we obtain

$$v_1 = -\frac{u\varepsilon_1 q P_1(q)}{s+uq} + \left[\frac{u\varepsilon_1 q P_1(q)}{s+uq} - \frac{\varepsilon_{11} q P_2(q)}{s+q}\right] \exp\left(-\sqrt{s+uq}\,\frac{z}{a}\right)\left(1+\frac{\sqrt{s+uq}}{\lambda a\sqrt{s+q}}\right)^{-1}; \tag{13}$$

$$v_2 = -\frac{\varepsilon_{11} q P_2(q)}{s+q} + \left[\frac{\varepsilon_{11} q P_2(q)}{s+q} - \frac{u\varepsilon_1 q P_1(q)}{s+uq}\right] \exp(-\sqrt{s+q}\,|z|)\left(1+\frac{\lambda a\sqrt{s+q}}{\sqrt{s+uq}}\right)^{-1}. \tag{14}$$

To check the solution obtained we will consider the special case $u = 0$, i.e., $u_1 = 0$ and $u_2 \neq 0$. In this case we obtain from (13) and (14)

$$v_1 = -\frac{\varepsilon_{11} q P_2(q)}{s+q} \frac{\exp\left(-\frac{z}{a}\sqrt{s}\right)}{1+\frac{\lambda a\sqrt{s+q}}{\sqrt{s}}};\; z>0, \tag{15}$$

$$v_2 = -\frac{\varepsilon_{11} q P_2(q)}{s+q}\left[1 - \frac{\exp(-\sqrt{s+q}\,|z|)}{1+\frac{\sqrt{s}}{\lambda a\sqrt{s+q}}}\right];\; z<0. \tag{16}$$

As might have been expected, Eqs. (15) and (16) are identical (apart from the notation) with the corresponding solutions of the problem of the thermal field close to the boundary of a throttling liquid given in [1].

It is difficult to obtain the originals of transforms (13) and (14). Hence, we will consider two important practical cases below.

To estimate the time taken for the thermal mode to build up, we will consider the case of identical rates of throttling of the fluids $u = 1$. From (13) and (14) we obtain

$$v_1 = -\frac{\varepsilon_1 q P_1(q)}{s+q} + \left[\frac{\varepsilon_1 q P_1(q)}{s+q} - \frac{\varepsilon_{11} q P_2(q)}{s+q}\right] \frac{\exp\left(-\sqrt{s+q}\,\frac{z}{a}\right)}{1+\frac{1}{\lambda a}}, \tag{17}$$

$$v_2 = -\frac{\varepsilon_{11} q P_2(q)}{s+q} + \left[\frac{\varepsilon_{11} q P_2(q)}{s+q}\right] \frac{\exp(-\sqrt{s+q}\,|z|)}{1+\lambda a}. \tag{18}$$

Using the operational relations in [2], we have after appropriate integration

$$T_1 = \begin{cases} -\varepsilon_1 [P_1(\varkappa) - P_1(\varkappa - \mathrm{Fo})] & \text{for } \varkappa > \mathrm{Fo} \\ -\varepsilon_1 P_1(\varkappa) & \text{for } \varkappa < \mathrm{Fo} \end{cases} + \frac{1}{1+\frac{1}{\lambda a}} \int_0^{\varkappa} \frac{\partial[\varepsilon_1 P_1(x) - \varepsilon_{11} P_2(x)]}{\partial x} \operatorname{erfc}\left(\frac{z}{2a\sqrt{\varkappa - x}}\right) I[\mathrm{Fo} - (\varkappa - x)]\,dx; \tag{19}$$

$$T_2 = \begin{cases} -\varepsilon_{11} [P_2(\varkappa) - P_2(\varkappa - \mathrm{Fo})] & \text{for } \varkappa > \mathrm{Fo} \\ -\varepsilon_{11} P_2(\varkappa) & \text{for } \varkappa < \mathrm{Fo} \end{cases} + \frac{1}{1+\lambda a} \int_0^{\varkappa} \frac{\partial[\varepsilon_{11} P_2(x) - \varepsilon_1 P_1(x)]}{\partial x} \operatorname{erfc}\left(\frac{z}{2\sqrt{\varkappa - x}}\right) I[\mathrm{Fo} - (\varkappa - x)]\,dx. \tag{20}$$

As can easily be shown from (19) and (20), the time taken for the temperature field to become established in the case considered for $z = 0$ corresponds to $\mathrm{Fo} = \varkappa$.

The steady-state temperature distribution $s = 0$ is given by the expressions

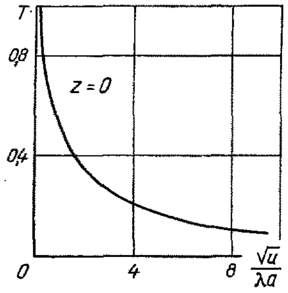

Fig. 1. Graph of T against $\sqrt{u}/\lambda a$ for z = 0.

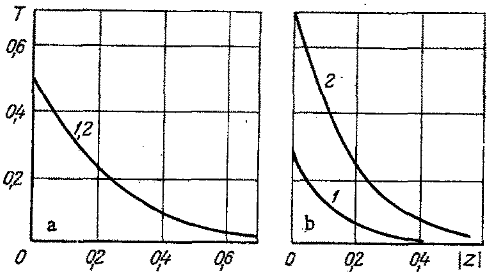

Fig. 2. Graphs of T against z. The values are calculated using the following equations: 1) $T = (T_1 + \varepsilon_I P_1(\varkappa))/(\varepsilon_I P_1(\varkappa) - \varepsilon_{II} P_2(\varkappa))$; 2) $T = (T_2 + \varepsilon_{II} P_2(\varkappa))/(\varepsilon_{II} P_2(\varkappa) - \varepsilon_I P_1(\varkappa))$; a) for $\lambda = a = u = 1$, b) $\lambda = a = u = 0.5$.

$$v_1 = -\varepsilon_I P_1(q) + [\varepsilon_I P_1(q) - \varepsilon_{II} P_2(q)]\,\frac{\exp\left(-\frac{\sqrt{u}\,z}{a}\sqrt{q}\right)}{1+\frac{\sqrt{u}}{\lambda q}}; \quad z>0, \tag{21}$$

$$v_2 = -\varepsilon_{II} P_2(q) + [\varepsilon_{II} P_2(q) - \varepsilon_I P_1(q)]\,\frac{\exp(-|z|\sqrt{q})}{1+\frac{\lambda a}{\sqrt{u}}}; \quad z<0. \tag{22}$$

The original of Eqs. (21) and (22) have the form

$$T_1 = -\varepsilon_I P_1(\varkappa) + \frac{1}{1+\frac{\sqrt{u}}{\lambda a}} \int_0^{\varkappa} \frac{\partial[\varepsilon_I P_1(x) - \varepsilon_{II} P_2(x)]}{\partial x} \operatorname{erfc}\left(\frac{\sqrt{u}\,z}{2a\sqrt{\varkappa - x}}\right) dx; \tag{23}$$

$$T_2 = -\varepsilon_{II} P_2(\varkappa) + \frac{1}{1+\frac{\lambda a}{\sqrt{u}}} \int_0^{\varkappa} \frac{\partial[\varepsilon_{II} P_2(x) - \varepsilon_I P_1(x)]}{\partial x} \operatorname{erfc}\left(\frac{z}{2\sqrt{\varkappa - x}}\right) dx. \tag{24}$$

For z = 0 we obtain from (23) and (24)

$$T = \frac{T_{1,2} - \varepsilon_I P_1(\varkappa)}{\varepsilon_I P_1(\varkappa) - \varepsilon_{II} P_2(\varkappa)} = \frac{1}{1+\frac{\sqrt{u}}{\lambda a}}. \tag{25}$$

The results of calculations of T as a function of the parameter $\sqrt{u}/\lambda a$ are shown in Fig. 1. In order to estimate the effect of heat transfer on the steady-state thermal field at the boundary of two throttling liquids, we will consider the following two cases:

a) the idealized case of the throttling of liquids with the same thermal and hydrodynamic properties, but different Joule–Thomson coefficients $\lambda_1 = \lambda_2$, $a_1 = a_2$, $u_1 = u_2$. Then, $T = 1/2$, i.e., the temperature of the boundary of the liquids is equal to the arithmetic mean of the temperatures of the throttling liquids ignoring heat transfer;

b) throttling of water and petroleum in sandstone [3]: $\lambda_1 = 1.7$ W/m·deg K, $a_1 = 11.6 \cdot 10^{-7}$ m²/sec, $\lambda_2 = 2.46$ W/m·deg K, $a_2 = 12.8 \cdot 10^{-7}$ m²/sec, and $u = 1/2$, we obtain T = 0.67.

To estimate the region of influence of heat transfer along the z axis we can use the results of calculations of T carried out on a computer using Eqs. (23) and (24) and represented in Figs. 2a and b.

NOTATION

t, time, T_1 and T_2, temperature in the regions $z > 0$ and $z < 0$, respectively; a_1, a_2, λ_1, and λ_2, thermal diffusivity and thermal conductivity in the corresponding regions; $\bar{x}$ and $\bar{z}$, coordinates; ε_I, ε_{II}, Joule–Thomson coefficients in the corresponding regions; P_1 and P_2, pressure distributions; R, characteristic length,

$$I(x)=\begin{cases}1; & x>0;\\ 0; & x<0;\end{cases}\quad \operatorname{erfc}(x)=\frac{2}{\sqrt{\pi}}\int\limits_{x}^{\infty}e^{-z^2}dz.$$

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APPLICATION OF INTEGRAL-RELATION METHOD IN USING COMPLEX MODELS OF TURBULENCE

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The generalization of the integral-relation method to the case when turbulence models with two differential equations for the turbulent flow properties is considered.

Recently, in achieving closure of the system of equations of turbulent liquid motion, there has been wide use of semiempirical theories of turbulence with one or more differential equations for the transfer of any turbulent flow properties [1-5]. Usually, the system of partial differential equations is numerically integrated, which requires considerable machine time.

In jet theory, at present, integral methods of solution are widely used [6]. One such is the integral-relation method, in which, rather than the initial system of partial differential equations, the solution for some integral relations obtained on the basis of this system is obtained. Solution by the integral-relation method rests on the similarity between the velocity, temperature, and concentration profiles in the jet, and reduces to integration of a system of ordinary differential equations. In a number of jet problems, the use of this method leads to very simple and clear relations.

Usually, integral relations are obtained on the basis of equations of motion, heat transfer, and impurities. The system of integral relations is then closed by the Prandtl formula (or another algebraic formula) for the tangential stress and its analogs for the heat transfer and impurities.

It is also expedient to use the integral-relation method when more complex models of turbulence – with one or more differential equations for any turbulent properties of the liquid – are used. Note that the literature includes a number of papers which use one integral relation obtained from the differential equation for the kinetic energy of turbulent pulsations. In these works, either the system of partial differential equations of motion and continuity is solved with this relation or this integral relation is solved for a single parameter and the other unknowns are determined from experiment [5, 7, 8].

Since more complex turbulence models contain new unknowns, it is necessary, accordingly, to generalize the integral-relation method so as to obtain new unknowns using integral relations derived on the basis of additional differential equations.

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